Place value, understanding 'ten of these is one of those', underpins fluency in arithmetic calculations and supports ongoing learning with larger numbers and decimals.

Place value

Why place value is important

Place value underpins fluency in arithmetic calculations and supports ongoing learning with larger numbers and decimals. 'Ten of these is one of those' is a key pattern underpinning initial place value understanding. Young learners can usually 'read' two-digit numbers before they actually understand that the placement of a digit in a number impacts its value. Learners may therefore be able to read 72 and 27 and state which one is larger/smaller, without knowing why the numbers have different values. An overemphasis on the mathematical recording of two-digit numbers undermines a learner's capacity for developing their mental and written computation strategies, particularly as our English number naming system does not support the place value pattern (eg eleven, twelve, the 'teens' as in thirteen, fourteen). Furthermore, many learners may consider 'twenty' as the name and '20' as the numeral for a collection of 20 objects. They are not necessarily connecting the '2' in '20' as 2 tens.

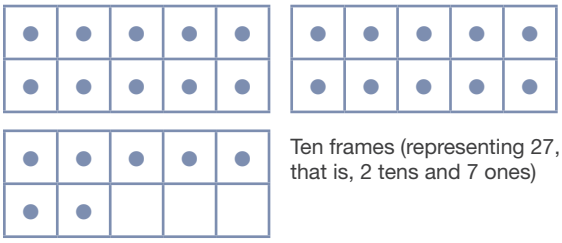
This paper provides guidelines on how an understanding of place value develops and how leaders can support educators in fostering this understanding, both in themselves and in their learners.

Developing understanding of place value

Place value represents an understanding that 1 is not necessarily just 1, but for the number 14 for example, the 1 represents 1 ten. This idea is difficult for young learners to grasp. The idea 'ten of these is one of those' is the building block of the base 10 numeration system and having a strong sense of numbers 0–10 is a pre-requisite for developing place value understanding. This means modelling, reading and writing the numbers to 10 using materials, words, diagrams and symbols as well as having mental objects for the numbers to ten (ie learners trust the count).

Learners should be able to *trust the count* and draw on mental models for each of the numbers without having to count by ones (see 3.1 Trusting the count).

Furthermore, counting large collections by 2s, 5s and 10s and recognising them as countable units is fundamental to establishing 10 as a countable unit. *Ten frames* can be used to prompt different mental models for numbers and different mental strategies for manipulating them in association with their relationship to 10. When appropriate, *ten frames* can also be used to represent larger numbers in terms of their tens part and their ones. The knowledge of place value patterns—that 10 of these is 1 of those—is necessary by the end of year 2 in order for learners to be able to work effectively with two-digit numbers and beyond.



Ten frames (representing 27, that is, 2 tens and 7 ones)

How can educators help learners understand place value?

Educators can provide regular opportunities for learners to make multiple bundles of ten (eg pop sticks) which can then be regularly counted. This can be achieved through activities, such as trading games, to consolidate the idea that 10 of these is 1 of those. For example, players take it in turns to toss a six sided dice and add the matching number of sticks to the ones place on a placemat. When the player reaches 10 ones these are bundled to make

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1 ten and transferred to the tens place on the same placemat. The game continues until the number made exactly equals a predetermined target number (eg 64). Alternatively, the game keeps going, and many bundles of tens are made. These can be counted in the following way, '1 ten, 2 tens, 3 tens, 4 tens, … 13 tens, 14 tens, … 24 tens, 25 tens. We made 25 tens!' Learners can then make, name and record tens and ones using a variety of materials. Place value charts can be used to model and record two-digit numbers.

As previously stated, the English names for the multiples of 10 are problematic (Boulton-Lewis & Halford, 1992; Nunes & Bryant, 1996; Verschaffel & De Corte, 1996). They should be dealt with as words before being introduced in symbolic form. The following order is suggested: sixty, seventy, eighty, ninety (as these names are based on the ordinal number names), then thirty and fifty (these are based on the ordinal number names), then twenty (should be two-ty) and forty (spelling is inconsistent with the spelling of four). It is recommended that then learners are ready to make/model, name and record numbers in the range 20–99 before dealing with the teen numbers more formally, even though learners would have seen these numbers recorded in the course of everyday events, such as writing the date. When students are sufficiently confident with the numbers from 20–99, thay are ready to make, name and record the teen numbers.

As learners become familiar with making/representing, naming and recording all the numbers to 99, their experiences can be consolidated by applying their knowledge to practical, everyday problems in four basic ways:

- **comparing (eg comparing two measures to determine which is heavier, longer etc and justifying how they know)**
- **ordering a set of numbers (eg the ages of family members, youngest to oldest) or by placing numbers strategically on a number line**
- **counting forwards and backwards in place value parts**
- **renaming numbers in terms of their parts, for example, for 57 can rename as 5 tens, 7 ones; 4 tens, 17 ones; … 57 ones.**

Before working with hundreds as a new place value part, it is important that learners develop a sense of numbers beyond 100. Pattern work involving number charts that emphasises the continuing count of tens beyond 100 may be useful. It is best to ensure that two-digit numeration is deeply understood before proceeding to three-digit numeration too quickly. Structured materials such as MAB or dot cards (Ellemore-Collins & Wright, 2011) are less cumbersome when working with these larger numbers. Making and counting hundreds is not as problematic as it was for the tens, because the number-naming sequence is now fully supportive of the pattern.

Teen numbers (eg 617) and internal zeros (eg 503) can prove tricky for some learners and so they still need plenty of opportunities to make, name and record hundreds, tens and ones using structured materials and place value parts, for example '*Make me 5 hundreds, 0 tens and 3 ones*'. Realistic opportunities to compare and order three-digit numbers and/or measures can be generated from newspaper articles and advertising brochures. Opportunities to rename numbers can arise through measurement, for example, using a measuring tape labelled with millimetres becomes a renaming task when the student is asked 'How many centimetres is this?'. If the table height is 760 mm, there are 76 tens, so the table is 76 cm high.

How can leaders support their staff?

It is important to support educators to design learning tasks that engage learners in productive exploration of the place value pattern. Such tasks will allow learners to develop connections between amounts together with the physical, oral and written representations of those amounts. When educators experience collaborative learning and exploration for themselves, they are more likely to successfully organise classroom learning in the same way. Educators can then develop forms of evidence and assessment that focus on the big idea of place value rather than moving on too quickly to superficial worksheet-based assessments. In addition to bundling and stacking material such as icy pole sticks and unifix blocks, resources, such as the ten frames and dot cards provided by the NRICH Project (University of Cambridge), can support educators in designing learning activities to develop conceptual understanding.

When observing lessons, look for learners who:

- can make connections between amounts and a variety of representations
- can model (eg using concrete materials), name, and record two-digit numbers—and later three-digit numbers—and explain place value to their peers or the teacher
- when talking about practical everyday problems, demonstrate their knowledge of place value by comparing, ordering, counting and renaming.

Leaders should consider how they work with their staff to incorporate the big ideas in number into common agreements around planning, teaching and assessment at their site.

Reflective questions for leaders to ask their teachers

When looking at and discussing the numeracy and mathematics program, you could, for example, ask the teacher:

- What evidence tells you that students in your classroom understand place value? How are you extending their learning?
- When learners are struggling to understand place value, what specific steps or activities prove useful?
- Have you used the place value common misunderstanding tools? (Siemon, 2009)
- How do you balance the need for learners to explore the concept of place value and your timing of explicit instruction?
- How do you use evidence of learner thinking about place value to inform your planning and assessment of learner achievement?

Further resources

The big ideas in number are discussed in further detail in the following mathematics papers:

- 3.0 Conceptual understanding: Number and algebra
- 3.1 Trusting the count
- 3.3 Multiplicative thinking
- 3.4 Partitioning
- 3.5 Proportional reasoning
- 3.6 Generalising.

All papers in this series are based on the work of Dianne Siemon, Professor of Mathematics Education at RMIT and a key text (Siemon et al, 2015).

http://bit.ly/BestAdviceSeries

Further reading

ACER PAT Teaching Resources Centre houses relevant concept builders, for example:

- place value (representing decimals)
- place value (whole numbers).

Van De Walle JA, Karp K & Bay-Williams JM (2016) *Elementary and Middle School Mathematics: Teaching Developmentally*, Ninth global edition, UK: Pearson Education Limited. In particular, refer to chapter on 'Algebraic thinking: Generalisations, patterns and functions'.

Victorian Department of Education and Training, Mathematics Developmental Continuum F–10 This resource provides evidence-based indicators of progress, linked to powerful teaching strategies.

Victorian Department of Education and Training, Assessment for Common Misunderstandings These tools draw on highly focussed, research-based Probe Tasks and the Probe Task Manual (RMIT), as well as a number of additional tasks and resources which have been organised to address 'common misunderstandings'.

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Ellemor-Collins D & Wright R (2011) 'Developing conceptual place value: Instructional design for intensive intervention', *Australian Journal of Learning Difficulties*, 16(1), 41–63, viewed at https://www.merga.net.au/documents/Ellmore-Collins_RP09.pdf

NRICH Project, University of Cambridge: https://nrich.maths.org

Nunes T & Bryant P (1996) *Children doing mathematics*, Oxford, UK: Blackwell

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Siemon D, Beswick K, Brady K, Clark J, Faragher R & Warren E (2015) *Teaching Mathematics: Foundations to Middle Years*, 2nd edition, Melbourne, Oxford University Press

Verschaffel L & De Corte E (1996) 'Number and arithmetic', in A Bishop, K Clements, C Keitel, J Kirkpatrick & C Laborde (Eds.) *International Handbook of Mathematics Education*, pp. 99–137, Dordrecht, Netherlands: Kluwer

This paper is part of the Department Leading Learning Improvement *Best advice* series, which aims to provide leaders with the research and resource tools to lead learning improvement across learning areas within their site.

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